FM-50 **Mechanics (Vectors – Rotational Mech.)** Time-1hr

Answer all.

1. i) State work-energy theorem. Prove it for a variable force.
   ii) Derive an expression for the potential energy stored in a system of a block attached to a massless spring of spring constant k when the block is pulled from its equilibrium position by x.
   iii) Draw the variation of potential energy & KE of the block attached to the spring which obeys Hooke's law. [5+3+2]

2) (i) Define Centre of Mass of a system. Find the centre of mass of a hemisphere of radius R. [1+2
   (ii) State and prove the principle of conservation of angular momentum. [3
   (iii) Find the moment of inertia of a sphere of mass M and radius R about one of its diameters. [4

II. $2\times 15 = 30$ (−1 for each wrong answer).

3) A physical quantity of the dimensions of length that can be formed out of c, G and $\frac{e^2}{4\pi\varepsilon_0}$ is (a) $c^2\left[\frac{Ge^2}{4\pi\varepsilon_0}\right]^{1/2}$ (b) $\frac{1}{c^2}\left[\frac{e^2}{G4\pi\varepsilon_0}\right]^{1/2}$ (c) $\frac{1}{c}G\frac{e^2}{4\pi\varepsilon_0}$ (d) $\frac{1}{c^2}\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2}$

4) The error in the measurement of radius of a sphere is 2%, then the error in the determination of volume of the sphere is
   (a) 8% (b) 2% (c) 4% (d) 6%

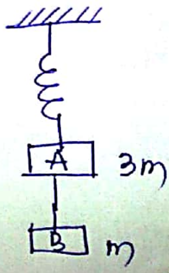
5) A particle moves a distance x in time t according to the equation $x = (t+5)^{-1}$. The acceleration of the particle is proportional to
   (a) $(\text{velocity})^{3/2}$ (b) $(\text{distance})^2$ (c) $(\text{distance})^{-2}$ (d) $(\text{velocity})^{2/3}$.

6) In the figure $a = 15\,m/s^2$ represents the total accel.^{n} of a particle moving in the clockwise dir.^{n} in a circle of radius $R = 2.5\,m$ at a given instant of time. The speed of the particle is (in m/s) (a) 4.5 (b) 5 (c) 5.7 (d) 6.2

7) The speed of a projectile at its maximum height is half its initial speed. The angle of projection is (a) 60° (b) 15° (c) 30° (d) 45°

8) The vectors $\vec{A}$ & $\vec{B}$ are such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle between the vectors is (a) 45° (b) 90° (c) 60° (d) 75°

9) Two blocks A & B of masses 3m & m respectively are connected by a massless, inextensible string. The whole system is suspended by a massless spring. The magnitudes of accl.^{n} of A & B just after the string is cut are –
   (a) $g/3$, g (b) g, g (c) $g/3$, $g/3$ (d) g, $g/3$



10) A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (2t\,\hat{i} + 3t^2\,\hat{j})$ N. Power developed at t is

(a) $(2t^3+3t^4)$ W (b) $(2t^3+3t^5)$ W (c) $(2t^2+3t^3)$ W (d) $(2t^2+4t^4)$ W

11) What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

(a) $\sqrt{3gR}$ (b) $\sqrt{5gR}$ (c) $\sqrt{gR}$ (d) $\sqrt{2gR}$

12) The PE between 2 atoms in a molecule is $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, $a, b > 0$. The atoms are in stable equilibrium when-

(a) $x = \left(\frac{2a}{b}\right)^{1/6}$ (b) $\left(\frac{11a}{5b}\right)^{1/6}$ (c) $x = 0$ (d) $x = \left(\frac{a}{2b}\right)^{1/6}$

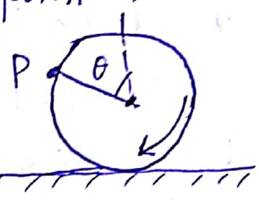
13) A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. The speed of the centre when it ceases to slip is -

(a) $r\omega_0$ (b) $\frac{r\omega_0}{4}$ (c) $r\omega_0/3$ (d) $r\omega_0/2$.

14) The moment of inertia of a uniform square plate of side a & mass m about an axis perpendicular to its plane & passing through one of its corners is (a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$ (c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$

15) A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled & KE is halved, the new angular momentum will be (a) $L/4$ (b) $2L$ (c) $4L$ (d) $L/2$.

16) A wheel is rolling straight on ground without slipping. If the axis of the wheel has speed v, the instantaneous velocity of a point P on the rim defined by angle θ relative to ground is



(a) $v\cos(\theta/2)$ (b) $2v\cos(\theta/2)$ (c) $v(1+\sin\theta)$ (d) $v(1+\cos\theta)$

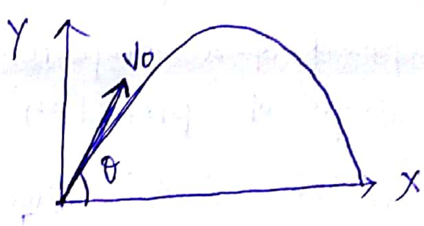
17) A small particle of mass m is projected at an angle θ with the x-axis with initial speed v_0 in the XY plane. At time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is

(a) $(-mgv_0t^2\cos\theta)\,\hat{j}$

(b) $mgv_0t\cos\theta\,\hat{k}$

(c) $-\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$

(d) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$



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